Pseudo-analysis as a tool of information processing

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The theory of the pseudo-analysis is based on the idea to replace the field of real numbers endowed the usual addition and product, with new operations so-called pseudo-addition and pseudo-multiplication in the framework of the semiring [10-16]. These operations are related to the process of combining several numerical values into a single representative, which is called aggregation, and the corresponding numerical function is called aggregation function, see [5,17]. Information fusion in an intelligent system is a fundamental problem, and its use is rapidly increasing as more complex systems are being developed, e.g., applied mathematics with probability, statistics, decision theory, computer sciences with artificial intelligence, operations research, as well as many applied fields as economy and finance, pattern recognition and image processing, data fusion, multi-criteria decision aid, automated reasoning, robotics, a fusion of images, integration of different kinds of knowledge, see [5,7,17].

The first traces of the pseudo-analysis goes to Grossman and Katz [6] and Burgin [2] (what today is called g-calculus, see [11]), then Maslov [8] (what today is called idempotent analysis). These results were managed as a complete unified theory later in the pseudo-analysis [9-16], and as a special case, the *g*-calculus [11]. The advantage of the pseudo-analysis is to enable to treat in a unified way three important problems as nonlinearity, uncertainty and optimization. Then corresponding pseudo additive measures and corresponding integrals were introduced. The usefulness of the pseudo-analysis is shown with some important applications in utility theory, nonlinear equations, fuzzy numbers, information theory, system theory, option pricing, large deviation principle, physics of the universe, see [1,3,4,7,8,12-16,18,19]. For example, pseudo-analysis is applied for solving nonlinear equations (ODE, PDE, difference equations, etc.) using the pseudo-superposition principle [8,12,14], which means that if $u1$ and $u2$ are solutions of the considered nonlinear equation, then also $a1 \otimes u1 \oplus a2 \otimes u2$ is a solution for any constants $a1$ and $a2$ from $[a, b]$. It was obtained an exact solution of the Burgers equation, as well as for the general Hamilton–Jacobi equations, where the nonlinear Hamiltonian is non-smooth, what is important in control theory. Another example is in the utility theory, which was earlier based on the notion of mathematical expectation in the axiomatic foundations by von Neumann and Morgenstern as probabilistic mixtures. The aim of the paper [4] was to extend maximally in a natural way the utility theory. The solution obtained in [4,15] is based on a result from [7,Th. 5.21], on the restricted distributivity of a t-norm over a t-conorm. An important consequence was that beyond possibilistic and probabilistic mixtures, only a hybridization is possible such that the mixture is possibilistic under a certain threshold, and probabilistic above.

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