Relations in conceptual spaces

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Concepts play an important role in human culture as the basic level of knowledge systems (Burgin, 2016). In essence, all meaningful words are names of concepts. Consequently, dictionaries are collections of concept definitions while encyclopedias are collections of extended concept descriptions. Thus, it is important to study concepts and their arrangements in the context of knowledge systems.

In terms of their organization, concepts form conceptual spaces (Doignon and Falmagne, 1999; Gärdenfors, 2004; Burgin, M. and Díaz-Nafría, 2019). The goal of this work is to study relations in conceptual spaces.

In conceptual spaces, concepts are represented by their models. In our study, we use the most advanced model, which is called the *representational model of concept* (Burgin and Gorsky, 1991). On its second level, it can be treated as a synthesis of the Russell's model of concept and Peirce's model of sign. In the representational model of concept, the name of the concept is connected to the concept representative, which consists of three components: denotat as the collection of all particular exemplifications or instantiations of this concept, meaning or connotation, and sense or intentionality.

Here we consider relations of two types: inner and intermediate relations in conceptual spaces (Burgin, 2012). On the first level of the space structure, *inner relations* of a conceptual space are relations between elements of this space, while *intermediate relations* of a conceptual space are relations between elements of this space and some other objects. Examples of intermediate relations are abstract properties.

It is possible to differentiate all concepts in a conceptual space into three groups – general, individual and impossible concepts, which are defined in the following way:

- A general concept has many instantiations.
- An individual concept has only one instantiation.
- An impossible concept does not have instantiations.

Note that the membership of a concept in one of these groups is contextual, i.e., it depends on the context. For instance, the name of a person can be an individual concept in one group where there are no other people with this name and a general concept in another group where there are several people with this name.

According to the type of the denotat, general concepts can be:

- Set concepts, in which the denotat is a set

- Class concepts, in which the denotat is a class
- Ensemble concepts, in which the denotat is a ensemble

In contrast to general sets and classes, elements in an ensemble are related to one another.

Note that each classification or typology of concepts determines intermediate relations in the conceptual space.

Several other intermediate relations as well as focal inner relations such as “to be more (less) general” or “to be more (less) abstract” are considered and explained. In addition, existential projections of concepts and conceptual projections of knowledge will be discussed. The aim is to explicate the structural organization of information in conceptual spaces.

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