Pollution information acquisition

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Pollution of water and air is an indispensable dilemma for contemporary society in general and many countries, including USA, in particular. The release from unknown sources such contaminants as bacteria, heavy metals, PFAS (per- and polyfluoroalkyl substances) and many others is an essential threat for the health of population. For instance, the bacterial contaminant release is the major reason of the beaches closing. Thus, there is the problem of finding pollution sources in rural and urban areas, or in other words, obtaining information about position of such pollution sources.

Naturally, there is a bulk of researches, in which the authors try to solve this problem (cf., for example, (Atmadja, et al, 2001; Benotti, et al, 2020; Yan, et al, 2019)). The methods, models and algorithms developed in this area belong to environmental forensics, which is the application of reliable scientific methods to address problems related to release histories and sources of contamination in the environment.

In this context, the goal of the presented research is the development of mathematical models for point pollutant sources detection in water and air pipe network systems. This problem, in particular, includes:

1. Detecting source of pollution in potable water systems.
2. Identifying smoke source in ventilation system.
3. Locating source of the explosive in air system.
4. Detecting pollution of water streams while monitoring multiple streams from remote locations.

To detect the point source of pollution, we use models of the pollution propagation. The problem of reconstructing the process of propagation of the pollution through water pipe network system is reduced to solving inverse problem for identification the coordinates of the origin of pollution and the mass rate injection at this point (Burgin and Dantsker, 2015).

In turn, this involves solving multiple operator equations in general and differential equations in particular. As a rule, these equations do not have analytical solutions and demand computational methods and algorithm for their solutions.

Existing algorithms in this area, which employ real numbers, do not give sufficiently precise solutions and we based our model on algorithms that utilize hypernumbers.

The theory of hypernumbers is an innovative mathematical theory, which essentially extends possibilities of solving operator and differential equations by computational and analytical methods (Burgin, 2010; 2012; Burgin and Dantsker, 1995).

Construction of real hypernumbers from real numbers is similar to construction of real numbers from rational numbers. Namely, real hypernumbers are classes of equivalent sequences of real numbers. Complex hypernumbers are constructed by a similar technique from complex numbers. Hypernumbers amplified the power of the calculus allowing researchers to find the sum of any series of real or complex numbers, to integrate any real function, to build a rigorous construction of the Feynman path integral, to solve a variety of operator equations and to solve some problems in probability theory. All these innovations well correlate with other extensions of conventional methods. For instance, series summation in hypernumbers expands summation methods based on asymptotic series and formal power series.

The model and corresponding algorithms developed by the authors are utilized for detecting the following characteristics:

- Distance from the sensor to the pollution source
- Rate of the pollutant discharge into the water
- Coefficient of dispersion

To conclude, we emphasize that this work is the first stage in the development of the Pollution Positioning System (PPS), which would be able to detect location and other characteristics of pollution sources with sufficiently high precision.

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