Arithmetic loophole in Bell's theorem: Overlooked threat to entangled-state quantum cryptography

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Bell's theorem is supposed to exclude all local hidden-variable models of quantum correlations. However, an explicit counterexample shows that a new class of local realistic models, based on generalized arithmetic and calculus, can exactly reconstruct rotationally symmetric quantum probabilities typical of two-electron singlet states. Observable probabilities are consistent with the usual arithmetic employed by macroscopic observers, but counterfactual aspects of Bell's theorem are sensitive to the choice of hidden-variable arithmetic and calculus. The model is classical in the sense of Einstein, Podolsky, Rosen, and Bell: elements of reality exist and probabilities are modeled by integrals of hidden-variable probability densities. Probability densities have a Clauser-Horne product form typical of local realistic theories. However, neither the product nor the integral nor the representation of rotations are the usual ones. The integral has all the standard properties but only with respect to the arithmetic that defines the product. Certain formal transformations of integral expressions one finds in the usual proofs a la Bell do not work, so standard Bell-type inequalities cannot be proved. The system we consider is deterministic, local-realistic, rotationally invariant, observers have free will, detectors are perfect, hence the system is free of all the canonical loopholes discussed in the literature.

References

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