

# Nonlinearities and Interference - their importance for the study of information

Annette Grathoff

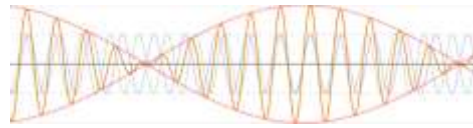
GSIS, 1220 Vienna, Austria; annette.grathoff@gsis.at; Design Institute, 80797 Munich, Germany

From a perspective of physical information – and it is proposed here that basic information is physical – mixing processes are important since they first allow the generation of novelty (new frequencies) and second mixing and other processes enabled by non-linear properties of wave carrying media in balance with the natural dispersive properties of waves can lead to the emergence of solitary waves and localized structures. Both effects – starting with an example for the emergence of stable structures in media, will be presented, showing the importance of the interference- and heterodyning dynamics for physical information processing from an evolutionary view. Solitons became known by an observation of Russell (1808–1882) in water (Union Canal system). The narrative of him observing the generation of a wave by a canal boat pulled by horses stopping suddenly, realizing the special character of the wave and then following its course over more than a mile on horseback is often told. After this observation Russell built his own “soliton generator” to further understand and analyze the interesting phenomenon. “Particle-like” behaviour gave the phenomenon its name. Localized pulses or “wavepackets” in general are maintaining their identities even when they undergo collisions with each other. But solitons are especially interesting for the study of information for yet another reason, namely for showing that particle-like behaviour of packets of wave- frequency components demands for a compensation of wave dispersion behaviour; compensation which here – in contrast to stability of atoms and elementary particles – appears to be a property of the medium in which the wave is excited at least as much as it is a property of the wave itself. A soliton cannot be viewed separate from its medium, it emerges – so to speak – context dependent. The understanding of the structural basis for the occurrence of solitons took a long time. Korteweg and de Vries realized that the phenomenon described by Russell required an unusually large amplitude and that the medium’s non-linearity must be grounded in a behaviour that is different to waves of different amplitudes. A balance between dispersion and non-linear differential acceleration or enhancement<sup>1</sup> of vibrational modes gives the solitary wave its long-term stability of form. Out of a forcing *steady* flow, a *continuous* and *periodic* sequence of solitary waves or solitons can be generated inside a medium with non-linear behaviour if the spatio-temporal coupling is right. Solitons are a stable solution of non-linear partial differential equations, but non-linearity in media as facilitator of non-linear mixing of waves are essential for their evolution. To better visualize the mathematics behind the difference made by non-linear mixing of waves compared to purely linear mixing, the example of audible beating when synchronously sounding two tuning forks slightly different in tune is chosen. From

---

<sup>1</sup> both acceleration and enhancement are meant to be understood in a neutral, analytical way, i.e. positive as well as negative.

two sinoid waves which are almost identical (called  $f_1$  and  $f_2$ ), new frequencies are generated and can possibly be perceived:



**Figure 1.** The superposition of two sine waves with similar frequencies ( $f_1$  and  $f_2$ ) (blue and green) linearly causes an interference frequency sinoid at the sum frequency  $(f_1 + f_2)/2$  (orange) and another sinoid which is the envelope of  $(f_1 + f_2)$  modulated at difference frequency  $(f_1 - f_2)^2$ . The Fourier transform contains only the two frequencies  $f_1$  and  $f_2$  (blue and green).



**Figure 2.** A non linear distortion happening inside a non-linear medium can generate a perceivable signal of the non-linear frequencies: (a) out of the beating between  $f_1$  and  $f_2$  under the influence of a non-linear distortion in the medium can emerge (b) the non-linear frequencies: The sum-frequency  $(f_1 + f_2)$  (dark blue) and the difference frequency  $(f_1 - f_2)$  (red) modulating its envelope. The modulated frequency  $(f_1 + f_2)$  is the carrier frequency of the modulating frequency. The Fourier transform contains only the two nonlinear frequencies  $(f_1 + f_2)$  and  $(f_1 - f_2)$ .

When the new difference-frequency between the two non-linearly mixed frequencies is slow enough, humans do not hear it as a tone, but as a periodic oscillation in sound amplitude of the sum-frequency tone. Since when above  $\sim 20$  Hz the human auditory threshold, the beating of the two tuning forks can be heard as a tone. There needs to be a non-linear process happening somewhere. Hermann von Helmholtz (1821–1894) associated location of the frequency perception on the human basilar membrane with corresponding sinusoidal frequencies. To explain the beating, von Helmholtz established the ‘distortion theory’, an aspect of theory of existence of “objective combination tones”. Modern textbooks under the impression of new experimental results often emphasize theories of purely “subjective combination tones”. Recent experiments by Lohri at the University of Music and Performing Arts Vienna nevertheless strongly suggest the objective reality of combination tones and lead to the conclusion that materials inside musical instruments with freely vibrating components could be acting as non-linear media. In a way it can be said that non-linear mixing between periodic frequencies is among the if not the most basic process in physical evolution in which a dynamical whole can be generated where ‘the whole is more than the sum of its parts’; but with the concession that the original “parts” cannot be recovered from the addition as soon as it developed into a non-linear mixing. The interesting question from an evolutionary perspective is how can mechanisms which “handle” waves of different amplitude or frequency in different ways be envisioned? How can mathematically representable non-linearities be physically acting in structures? A lot has yet to be discovered, preferably in an interdisciplinary scientific research venture.

<sup>2</sup> For those who wonder why the difference frequency is not given as  $(f_1 - f_2)/2$ : Since for the audible envelope beating it is irrelevant whether amplitude develops above or below the baseline, the frequency of the loudness fluctuation is effectively doubled  $f_{beat} = |f_1 - f_2|$ .