Vagueness and Errors

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It is undeniable that information flow is crucial for the dissemination of information and knowledge, which can be considered as information plus analysis and/or experience. The mathematical representation of information is quite a challenging task. The representation introduced by Barwise and Seligman [1] is quite interesting and useful. It employs the notion of a *type*. Although many readers would be familiar with this word, still we suspect few of them can really explain the difference between a set and a type.

Herman Geuvers [2] has elegantly explained the difference, and we borrow his explanation. First of all, a type is not a set. Types may look like sets but give syntactic information. For example, in the expression$4/x + 2{:}\, nat$ the word nat says that expression is an integer, meaning that all constituents are integer numbers, operations, and variables. Now consider the expression $8 \in \{\,\mathrm{x} \mid \mathrm{x} = 2n, where\ n \in \mathbb{N}\}$. This states that 8 is an even number. More generally, to say that 8 is an even number we need a *proof* that will establish that there is a natural number such that when multiplied by 2 it is equal to 8. To show that $4/x + 2{:}\, nat$, we need a computation that will make sure that all terms of the expression arenat

The basic ingredient of the representation of Barwise and Seligman is the classification:

Definition 1. *A classification* $\boldsymbol{A} = (A, \Sigma_A, \vDash_A)$ consists of a set A of objects tobe classified, called *tokens* of $\boldsymbol{A}$, a set $\Sigma_A$of objects used to classify the tokens, called the *types* of $\boldsymbol{A}$, and a binary relation $\vDash_A$ over A andΣ_A that tells one which tokens are classified as being of which type.

In simple words, we have set of types, a set of objects (the tokens), and a relation that tells us what is the type of each object. Thus $a \vDash_A \beta$ means that the token a is of type β.

Example 1. Suppose we have a set B of batteries and Σ_B is a set that includes the types *empty, full*, etc. The relation $\vDash_B$ can be used to classify the batteries. Thus $b_1 \vDash_B empty$ means that battery b_1 is *empty*.

Functions are used to map numbers to numbers or, more generally, objects to objects. An *informorphism* is "function" that maps one classification to another. Thus if one classification is about flashlight bulbs and their status and another about batteries and their status, then an infomorphism maps flashlight bulbs and their status to batteries and their status. Formally:

Definition 2. If $\boldsymbol{A} = (A, \Sigma_A, \vDash_A)$ and $\boldsymbol{B} = (B, \Sigma_B, \vDash_B)$ are classifications, then an *infomorphism* is a pair $f = (f^{\rightarrow}, f^{\leftarrow})$ of functions, where $f^{\rightarrow}: B \rightarrow A$and $f^{\leftarrow}: \Sigma_A \rightarrow \Sigma_B$ are functions, satisfying the condition:$f^{\rightarrow}(b) \vDash_A \alpha \Leftrightarrow b \vDash_B f^{\leftarrow}(\alpha)$, for all tokens b of $\boldsymbol{B}$ and all types α of $\boldsymbol{A}$.

Based on earlier work by de Paiva, Papadopoulos and Syropoulos [3] introduced fuzzy classifications but they did not study them in the context of information flow. Their definition follows.

Definition 3. A *fuzzy classification* $\boldsymbol{A} = (A, \Sigma_A, \vDash_A)$ consists of a set A of objects to be classified, called *tokens* of $\boldsymbol{A}$, a set $\Sigma_A$of objects used to classify the tokens, called the *types* of $\boldsymbol{A}$, and a fuzzy binary relation , $\vDash_A$ over A and$\Sigma_A$such that$\vDash_A (b, \alpha) = i$ means that the object b is classified as being of type $\alpha$with degree that is equal to i.

Fuzzy informorphisms are a bit different:

Definition 4. If $\boldsymbol{A} = (A, \Sigma_A, \vDash_A)$ and $\boldsymbol{B} = (B, \Sigma_B, \vDash_B)$ are fuzzy classifications, then a *fuzzy infomorphism* is a pair $f = (f^{\rightarrow}, f^{\leftarrow})$ of functions, where $f^{\rightarrow}: B \rightarrow A$ and $f^{\leftarrow}: \Sigma_A \rightarrow \Sigma_B$ are functions, satisfying the condition:$\vDash_A \left(\alpha, f^{\rightarrow}(b)\right) \leq \vDash_B (f^{\leftarrow}(\alpha), b)$, for all tokens b of $\boldsymbol{B}$ and all types α of $\boldsymbol{A}$, and the relation $\leq$ is the usual order relation between numbers.

Ordinary classification is based on the assumption that nothing is lost during information flow, however, with fuzzy classifications I introduce the idea that something is actually lost during information flow. We all know (?) that errors occur during data transmission but here I will try to put forth the idea that these errors are due to vagueness. And the fuzzy classifications are a good tool to describe how vagueness affects information flow.

References

[1] Barwise, J., and Seligman, J. Information Flow: The Logic of Distributed Systems. Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, Cambridge, UK, 1997.

[2] Geuvers, H. Introduction to Type Theory. In Language Engineering and Rigorous Software Development: International LerNet ALFA Summer School 2008, Piriapolis, Uruguay, February 24–March 1, 2008, Revised Tutorial Lectures, A. Bove, L. S. Barbosa, A. Pardo, and J. S. Pinto, Eds. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009, pp. 1–56.

[3] Papadopoulos, B. K., and Syropoulos, A. Categorical relationships between Goguen sets and “two-sided” categorical models of linear logic. Fuzzy Sets and Systems 149 (2005), 501–508.