Expected emergent open-endedness from partial structures extensions under algorithmic perturbations

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Enactivism claims that the interaction in the environment brings forth a world, where the changes into the organisms that are emergent with respect to the environmental surroundings occur at the same time that the organisms are capable of bringing forth an environment that is emergent with respect to the respective embedded organism [1]. In complex systems science, the dependency on the observer’s a priori knowledge constitutes a central problem for delimiting what is an emergent phenomenon from what it is not. Thus, such a *two-way co-emergence* in enacting systems may suffer from the same fundamental dependency of the observer’s knowledge about the system’s behavior. In this sense, within the theoretical framework of model-theoretic systemics [2], algorithmic information dynamics (AID) [3], and algorithmic networks (ANs) [4,5], this work presents an existence proof of two-way co-emergent phenomena that are asymptotically independent of observers. In the particular models we present in this work, we show that these emergent phenomena assume the form of irreducible information not only about the dynamical systems’ states trajectories but also about the relational (partial) structures [6] that underlie and define each system’s functioning, which becomes a stronger form of observer-invariant emergence. Our results connect the unbounded increase of emergent algorithmic information in complex systems with endless irreducible extensions of formal knowledge.

In our models, each system A is a (finite discrete deterministic) dynamical system (DS), where its evolution rule (ER) is denoted by $S_A(A_t , E_{A,t} , t)$ [7]. Any population of DSs is contained into a single environment E, where the environment E is a discrete dynamical system (which is determined by the ER $\boldsymbol{E}(E_t , t)$) from which every DS is a sub-state, i.e., $S_A(A_t , E_{A,t} , t)$ is contained in $\boldsymbol{E}(E_t , t)$ for every system A and time instant t. In addition, $\boldsymbol{E}$ may be a function at a higher computational class than the ERs of its DSs. Each environmental surrounding $E_{A,t}$ of a system A is composed of other systems with which A can interact. A system A interacting with a system B is understood as: A algorithmically perturbing [3] the states of B at time instant t_i so that, after the perturbation, $S_B(B_t , E_{B,t} , t)$ can be replaced with another evolution rule $S'_B(B'_t , E_{B,t} , t)$ for every $t+1>t_i$.

One can immediately prove that, for every possible sequence $A_1 ,..., A_{t'}$ of the t' states of a fixed ER S_A, there is a partial (relational) structure $\boldsymbol{S}_{A,t'}=(D_i , R_i)_{i \in I}$ [6] that univocally determines A until t', where D_i is the universe set of the partial relation $R_i=<R_{i1} , R_{i2} , R_{i3}>$. In this particular case of DSs, we have that $|I|=1$ and: R_{i1} is the set of 4-tuples that are *true* functional relations $A_{t+1}=S_A(A_t , E_{A,t} , t)$ with $t<t'$; R_{i2} is the set of 4-tuples that are *false* functional relations $A_{t+1}=S_A(A_t , E_{A,t} , t)$ with $t<t'$; and R_{i3} is the set of 4-tuples for which the function values $A_{t+1}=S_A(A_t , E_{A,t} , t)$, where $t+1>t'$, are *unknown*. Now, define the fitness value of a DS A until time instant t' to be the total number of combinations of possible internal states A_t and possible environmental surroundings $E_{A,t}$ in every possible state space trajectory of A of length $t<t'+1$ that does not lead A to degeneration (or “death”). In this

way, the larger the fitness of a system, the larger the number of environmental conditions that it can survive to. Since every system *A* is a finite discrete dynamical system, one can always computably associate each $\boldsymbol{S}_{A,t'}$ with a unique halting program that returns the fitness value of *A*. As a consequence, an AN studied in [4,5] becomes Turing equivalent to a particular type of population of DSs in which systems can perturb their (outgoing) neighbors. In this case, as the population increases in size, the expected algorithmic information necessary for computing the *networked* behavior of a system *A* or its environmental surroundings is boundlessly larger than the expected algorithmic information necessary for computing the *isolated* behavior of either system *A* or its environmental surroundings. Moreover, the expected algorithmic information necessary for computing the encoded atomic diagram of an isomorphic copy of $\boldsymbol{S}^{n}_{A,t_n}$ (which is the partial structure that determines the *networked* behavior) is unboundedly larger than the one for $\boldsymbol{S}^{1}_{A,t_n}$ (which is the partial structure that determines the *isolated* behavior). In other words, the sequence $(\boldsymbol{S}^{1}_{A,t_1}, \ldots, \boldsymbol{S}^{n}_{A,t_n})$ of nested substructures (i.e., the sequence that determines the networked behavior) extends the partial structure $\boldsymbol{S}^{1}_{A,t_1}$ in such a way that the embeddings leading to $\boldsymbol{S}^{n}_{A,t_n}$ are expected to be irreducible to $\boldsymbol{S}^{1}_{A,t_n}$, which in turn implies that, in addition to being organized, a DS determined by $\boldsymbol{S}^{n}_{A,t_n}$ is necessarily self-organized [2].

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